

RESEARCH PROBLEMS

Problem 102. Posed by John Reay.

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Theorem (See [3] for a proof and definitions). *Let $S \subset R^d$ be a set of at least $(d+1)(r-1) + (k+1)$ strongly independent points, where $0 \leq k \leq d$. Then S has a partition $S = S \cup \dots \cup S_r$ so that the k -dimensional volume $V(k) = \text{vol}_k[\bigcap_{i=1}^r \text{conv } S_i]$ satisfies $V(k) > 0$.*

If $(r, k) = (2, 0)$ this is Radon's theorem; $(r, k) = (r, 0)$ is Tverberg's theorem.

Problem. Find a suitable independence condition for S to assure $V(k) > 1$. See [1] and [2] for a similar Helly-type results.

References

- [1] I. Barany, M. Katchalski and J. Pach, Helly's theorem with volumes, Amer. Math. Monthly, vol. 91, No. 6 (1984) 362–365.
- [2] I. Barany, M. Katchalski and J. Pach, Quantitative Helly-type theorems, Proc. Amer. Math. Soc., vol. 86, No. 1 (1982) 109–114.
- [3] J. Reay, An extension of Radon's theorem, Illinois J. Math., vol. 12, No. 2 (1968) 184–189.

Problem 103. Posed by Robert A. Melter.

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Define the elongation of a rectangle to be the ratio of the length of its longer side to the length of its shorter side. Consider a polygon P with lattice points as vertices and sides parallel to the coordinate axes. For a partition Π of P into disjoint rectangles, let $S(\Pi)$ denote the sum of the elongations of the rectangles. Take $\mu(P)$ to be the minimum of $\mu(\Pi)$ over all partitions of P . Find bounds and/or formulas for $\mu(P)$ and describe the minimizing partitions.